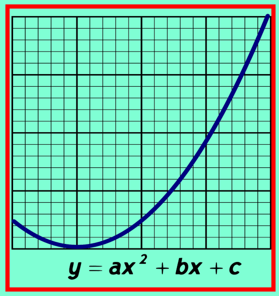
Math 125

Fall 2021

Lecture 26



Matrix: Array or table of numbers.

It has rows & It has columns

Use Brackets for matrix notation.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ -3 & 1 & 4 \end{bmatrix}$$

2 Rows

3 Columns

2×3

Matrix A is 2 by 3.

$$B = \begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 0 & 4 \end{bmatrix}$$

3 Rows

2 Columns

Matrix B is 3 by 2

3×2

Row matrix has only one row.

$$C = \begin{bmatrix} 1 & 2 & -3 & 5 \end{bmatrix}$$

1×4

Column matrix has only one Column.

$$A = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

3×1

Square Matrix: # Rows = # columns

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & -5 \\ 0 & 0 & 2 \end{bmatrix}$$

2×2 3×3

Determinant: It is a numerical value associated with a square matrix.

use | | to display determinant.

$$A = \begin{bmatrix} & \end{bmatrix} \qquad |A| = \begin{vmatrix} & \end{vmatrix}$$

Matrix A Det(A)

Det(A) can be positive, negative, or Zero.

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How to find the determinant of a 2x2 Matrix.

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \qquad |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

ex: Find $\begin{vmatrix} 4 & 3 \\ 1 & -2 \end{vmatrix} = 4(-2) - 1(3)$

$= -8 - 3 = \boxed{-11}$

Find $\begin{vmatrix} -3 & 5 \\ 6 & -10 \end{vmatrix} = (-3)(-10) - 6(5)$

$= 30 - 30 = \boxed{0}$

Evaluate $\begin{vmatrix} 5 & -2 \\ 0 & 4 \end{vmatrix} = 5(4) - 0(-2)$

$= 20 - 0 = \boxed{20}$

Cramer's Rule

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{D_y}{D}$$

This method only works when $D \neq 0$

$$\begin{cases} 5x - 4y = 2 \\ 6x - 5y = 1 \end{cases}$$

$D = \begin{vmatrix} 5 & -4 \\ 6 & -5 \end{vmatrix} = 5(-5) - 6(-4)$

$= -25 + 24 = \boxed{-1}$

$D_x = \begin{vmatrix} 2 & -4 \\ 1 & -5 \end{vmatrix} = 2(-5) - 1(-4)$

$= -10 + 4$

$= \boxed{-6}$

$D_y = \begin{vmatrix} 5 & 2 \\ 6 & 1 \end{vmatrix} = 5(1) - 6(2)$

$= 5 - 12$

$= \boxed{-7}$

$x = \frac{D_x}{D} = \frac{-6}{-1} = \boxed{6}$

$y = \frac{D_y}{D} = \frac{-7}{-1} = \boxed{7}$

$\Rightarrow \boxed{(x, y) = (6, 7)} \Rightarrow \{(6, 7)\}$

Solve by Cramer's Rule:

$$\begin{cases} 5x + 4y = 12 \\ 3x - 6y = 24 \end{cases}$$

$$D = \begin{vmatrix} 5 & 4 \\ 3 & -6 \end{vmatrix} = 5(-6) - 3(4) = -30 - 12 = \boxed{-42}$$

$$D_x = \begin{vmatrix} 12 & 4 \\ 24 & -6 \end{vmatrix} = 12(-6) - 24(4) = -72 - 96 = \boxed{-168}$$

$$D_y = \begin{vmatrix} 5 & 12 \\ 3 & 24 \end{vmatrix} = 5(24) - 3(12) = 120 - 36 = \boxed{84}$$

$$x = \frac{D_x}{D} = \frac{-168}{-42} = \boxed{4}$$

$$y = \frac{D_y}{D} = \frac{84}{-42} = \boxed{-2}$$

$$\Rightarrow \boxed{(x, y) = (4, -2)}$$

Sum of two numbers (x, y) is 50.

their difference is 10.

Use Cramer's rule to find them.

$$\begin{cases} x + y = 50 \\ x - y = 10 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(-1) - 1(1) = \boxed{-2}$$

$$D_x = \begin{vmatrix} 50 & 1 \\ 10 & -1 \end{vmatrix} = 50(-1) - 10(1) = \boxed{-60}$$

$$D_y = \begin{vmatrix} 1 & 50 \\ 1 & 10 \end{vmatrix} = 1(10) - 1(50) = \boxed{-40}$$

$$x = \frac{D_x}{D} = \frac{-60}{-2} = \boxed{30}, \quad y = \frac{D_y}{D} = \frac{-40}{-2} = \boxed{20}$$

Two numbers are 30 & 20.

John has 15 coins.
Nickels & Dimes only.
Total Value: $1
How many of each?
Use Cramer's rule.

$x \to$ Nickels
$y \to$ Dimes

Convert to Cents

$$\begin{cases} x + y = 15 \\ 5x + 10y = 100 \end{cases} \quad \div 5$$

$$\begin{cases} x + y = 15 \\ x + 2y = 20 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1(2) - 1(1) = \boxed{1}$$

$$D_x = \begin{vmatrix} 15 & 1 \\ 20 & 2 \end{vmatrix} = 15(2) - 20(1) = \boxed{10}$$

$$D_y = \begin{vmatrix} 1 & 15 \\ 1 & 20 \end{vmatrix} = 1(20) - 1(15) = \boxed{5}$$

10 Nickels & 5 Dimes